The Effect Of Income Inequality On Import Prices[1]

Gina Pieters

Abstract

It has been empirically established that higher within-country income inequality in a destination (1) decreases the unit-value its imports, and impacts both (2) the value of trade and (3) the price mark-up of identical goods, with both positive and negative impacts documented. I propose the first model that can replicate these three observations: a monopolistic competition model of trade with income-based price discrimination and pricing to market. The main mechanism is the consumer's ability to endogenously select the variety of goods they wish to consume: high inequality causes higher income consumers to choose a larger variety at higher prices, while low income consumers a smaller variety at lower prices. When simulated, the model captures 50% of the observed relationship between income inequality and trade value (a simulated elasticity of 0.04 instead 0.09), but overestimates the magnitude of unit value response (-0.02 instead of -0.01).

1. Introduction

The resurgence of non-homothetic preferences in trade models, a literature summarized in Markusen (2013), has changed our understanding of trade prices: Non-homothetic preferences allow prices to depend on an importers income per capita, in addition to aggregate income. Surprisingly however, models that can capture the relationship between prices and income per capita struggle when applied to prices and income distribution.

I will target and replicate three more specific empirical findings in this paper. The first of these is that firm-level datasets reveal prices of identical products vary across destinations based upon inequality, although the direction of this dependence is not consistent. Flach and Janeba (2017) (wine) find that prices increase for countries with higher inequality while Simonovska (2015) (clothing) find that prices decrease. As the goods in these studies are identical across countries, it implies that the relationship between inequality and pricing is not driven solely by quality upgrading across destination markets. The model I introduce will focus on this pricing-to-market channel and can reproduce the lack of a stable relationship through its reliance on multiple economic parameters.

The second empirical fact I target is that unit-values decrease with income inequality, as shown in Lipsey and Swedenborg (2010) and Bekkers, Francois, and Manchin (2012). Finally, the third fact I repliace is the finding of Hummels and Lee (2017), who show that trade is strongly sensitive to changes in the tails of the income distribution — all else equal, changes in inequality should effect trade flows. Again, the direction is not consistent: Francois and Kaplan (1996) show that inequality increases demand for differentiated goods and therefore trade should increase, while Dalgin, Trindade, and Mitra (2008) show that inequality increases the imports of luxuries and decreases the imports of necessities. Adam, Katsimi, and Moutos (2012) show that inequality increases imports for high-income countries and decreases it for low-income countries. Eppinger and Felbermayr (2015) have verified that similar income distributions display similar import distributions and higher bilateral trade flows respectively, echoing the results of Choi, Hummels, and Xiang (2009) who show that countries with similar income distributions have similar import price distributions.

The paper most similar to mine is Bekkers, Francois, and Manchin (2012) who test the ability three commonly used non-homothetic demand structures to see if they can generate the negative relationship between income distribution and unit values. After testing variant of Stone-Geary-based hierarchic demand based on (Simonovska, 2015), ideal-variety demand (Fajgelbaum, Grossman, and Helpman, 2011; Hummels and Lugovskyy, 2009), and product-quality demand (Francois and Kaplan, 1996; Hallak, 2006) they find that only Stone-Geary-based hierarchic demand systems can correctly generate the negative unit value relationship with income distribution, implying that price mark-ups, not product quality, determine the relationship with unit value and inequality.[2]

However, the model used in Bekkers et al. (2012) is not empirically tractable, and I show in this paper that the simplest extension of Simonovska (2015) incorrectly predicts a positive unit value and no relationship between trade flows and income distributions, contradicting both empirical fact (ii) Negative unit value relationships, and (iii) impact on trade flows.

I introduce a general equilibrium heterogeneous-firm model of trade that incorporates within-country income-based price discrimination across consumers with identical non-homothetic preferences but different incomes in N countries in section 2. This model is the first to successfully generate the empirical relationships between trade value, unit value, and firm-level prices with both average income, income distribution, and population while also generating a tractable gravity equation that can be used to take the model to the data.

The model generates bounded marginal utility for goods so that a choke price exists above which consumers are unwilling to buy a good, even if they have market access. Consumers therefore endogenously choose both the quantity and variety of goods to consume. Heterogeneous firms are monopolistic price setters, and must choose whether to make their good available at a low price (such that both consumer-groups can afford the good), or a high price

[2] Simonovska (2015) show that firms charge higher prices for countries with higher income per capita for the exact same good, showing that pure mark-up behaviour can explain the positive relationship between price and income per capita, without a reliance on good quality.

(such that it is exclusively affordable to wealthy consumers). Firms can also potentially price discriminate between income groups, charging a higher price to high income consumers, up to an arbitrage constraint. The model contains two opposing forces. Firstly, more inequality means that high income earners earn higher incomes compared to their equivalents in a more equal economy, increasing their demand and ability to pay for goods. Simultaneously however, more inequality means that low income consumers in a less equal country demand fewer goods. A priori, either force can dominate in the market, allowing the model to reconcile the inconsistent, but statistically significant, effects of the aforementioned studies.

Section 5 shows how these forces affect individual firms, while section 3 show how these forces impact aggregate data. Simulation of the model in section 4 shows that, consistent with the data, the effects of the low income group dominates in unit value, while high income group dominates for value, capturing the documented relationship between inequality, import value, unit value with a degree of estimated price discrimination is similar to that found using USA grocery store scanner data. Additionally, the comparison of prices of identical goods across countries can be found to either increase or decrease across destinations depending on exclusivity of the good. Section 6 concludes.

2. Model

I consider a static environment with a finite number of countries, i $\in \{1,...,I\}$, trading varieties of final goods purchased by consumers. Let s denote the exporter ("source") and d the importer ("destination"). Each country has two consumer types, $k = \{H, L\}$, of population size M_d^k such that total population $M_d = \sum_k M_d^k$.

I follow Melitz (2003) and assume a continuum of monopolistic price-setting firms in each country, with the set of all firms in a country denoted by J_s. Firms are differentiated only by their productivity, z, and each produces using a labor-only production technology. Productivity and country uniquely identifies each firm. Each firm sells only one good, and goods from different countries are not substitutable, therefore a good from country s and firm z from country s are equivalent.

Consumer types differ only by their labor productivity, θ_d^k, which is fixed regardless of which firm employs them. They supply 1 unit of labor inelastically in the competitive labor market, so that effective labor supply is $L_d = M_d^H \theta_d^H + M_d^L \theta_d^L$ and type-k income is $w_d^k = \theta_d^k w_d$, where w_d is the general equilibrium wage per unit of labor productivity. As $w_d^H = \frac{\theta_d^H}{\theta_d^L} w_d^L$, the assumption that $\theta_d^H > \theta_d^L$ is sufficient to ensure that H-type has higher income than L-type.

2.1. Consumer's Problem

All consumers have the same utility function, and both consumer types within a country have access to identical goods. Given income, w_d^k, and prices, $p_{sd}^k(z)$, each consumer type chooses $c_{sd}^k(z)$ to solve

$$
\begin{aligned}
U_d^k = \max_{c_{sd}^k(z)} \sum_s \int_z \log(c_{sd}^k(z) + q)dz \\
\text{s.t} \sum_s \int_z p_{sd}^k(z) c_{sd}^k(z) dz \leq w_d^k \\
c_{sd}^k(z) \geq 0
\end{aligned}
\tag{1}
$$

where the non-homothetic parameter $q > 0$ is the same across all goods and countries. The solution is

$$
c_{sd}^k(z) = q \max\{0, \frac{Q_d^k}{p_{sd}^k(z)} - 1\} \tag{2}
$$

where for each consumer type-k

$$
Q_d^k = \frac{w_d^k + q P_d^k}{q N_d^k} \quad \text{(the cutoff price)} \tag{3}
$$

$$
N_d^k = \sum_s \int_z 1 dz \quad \text{(the variety of goods purchased)} \tag{4}
$$

$$
P_d^k = \sum_s \int_z p_{sd}^k(z) dz \quad \text{(the price aggregator)} \tag{5}
$$

Detailed algebraic derivations for all equations can be found in the accompanying online appendix. Equation 2 shows the non-homothetic parameter generates a bounded marginal utility for every good: if $p^k_{sd}(z) > Q^k_d$ the price for a specific good is "too high" and the consumer will choose the optimal boundary solution of $c^k_{sd}(z) = 0$. As the value of q increases, the highest price that the consumer will tolerate, Q^k_d, decreases. The value of q will be discussed during calibration in section 4.

The bounded marginal utility of consumers means that each consumer will not purchase all goods available from country s: $N^k_{sd} \subseteq J_s$. I will show that as w^k_d increases, Q^k_d will increase — higher income consumers tolerate a higher price before choosing a zero quantity. This endogenously generates the result that high income consumers in any country consume a larger variety of the goods than the low income group.

2.2. Firms Problem

Firms pay a tariff cost, $\tau_{sd} \geq 1$, where $\forall d \quad \tau_{dd} = 1$. Melitz-style models usually incorporate a fixed cost of exporting to generate the result that only a subset of domestic firms export. In my model, the consumer's cutoff price generates this result even if the export cost is zero. Given that the inclusion of a fixed export cost greatly increases the complexity of the model it is omitted.

The assumption of a linear production technology ensures that $w^H_d = w_d \theta^H_d$ and $w^L_d = w_d \theta^L_d$ so that firms are indifferent between the two types of labor. I therefore indicate the effective labor hired by a firm in s to produce goods for d as $\ell_{sd}(z) = \theta^H_s \ell^H_{sd}(z) + \theta^L_s \ell^L_{sd}(z)$. Market clearing imposes that $\sum_d \int_z \ell_{sd}(z) dz = L_s$.

Given the consumer's indirect demand functions $x(p_{sd}(z); P^k_d, N^k_d, w^k_d) = M^k_d c^k_{sd}(z)$, henceforth abbreviated $x^k_{sd}(z)$, firm z chooses the profit-maximizing action from four scenarios,

$$\pi_{sd}(z) = \max \quad \{\pi^H_{sd}(z), \pi^L_{sd}(z), \pi^B_{sd}(z), 0\} \tag{6}$$

where $\pi^k_{sd}(z)$ indicates that the firm sells exclusively to the k-type, π^B_{sd} indicates that the firm sells to both types, and 0 indicates shutdown (no sales).

2.2.1. *Selling to only a single consumer type:* $\pi_{sd}^H(z)$, $\pi_{sd}^L(z)$

If the firm sells only to consumer type k, it solves

$$\pi_{sd}^k(z) = \max_{p_{sd}^k(z),\ell_{sd}(z)} p_{sd}^k(z)x_{sd}^k(z) - w_s\ell_{sd}(z) \tag{7}$$

The optimal price is

$$p_{sd}^k(z) = \underbrace{\left[\frac{\tau_{sd}w_s}{z}\right]^{\frac{1}{2}}}_{\text{Marginal Cost}} \underbrace{Q_d^{k\frac{1}{2}}}_{\text{Mark-up}} \tag{8}$$

All firms charge the same mark-up to destination d and group k. The minimum productivity required to sell to group k, z_{sd}^k, found by solving $\pi_{sd}^k(z_{sd}^k) = 0$, is

$$z_{sd}^k = \frac{\tau_{sd}w_s}{Q_d^k} \tag{9}$$

A higher cutoff price allows less productive (lower z) firms to enter the market, while a higher trade barrier or cost of production requires firms to be more efficient to enter.[3]

Lemma 2.1. *If $Q_d^H > Q_d^L$ firms will never sell to only the L-type consumers d, $z_d^H < z_d^L$.*

This follows directly from equation (9), and implies that the firm is actually choosing from only three scenarios: sell only to rich consumers, sell to both, or shut down.

$$\pi_{sd}(z) = \max \quad \{\pi_{sd}^H(z), \pi_{sd}^B(z), 0\} \tag{10}$$

[3]The model predicts that predicts that $z_{sd}^k < z_{dd}^k$ if $\tau_{sd}w_s < w_d$ — a country with relatively low wages facing low export barriers will contain domestic firms that produce for exports that do not sell in the domestic market. This prediction does not exist for a model with fixed cost to export. Testing this prediction requires firm-level data from a low wage, low tariff country, and the authors are unaware of the existence of any such data-set.

2.2.2. *Selling to both consumer types:* $\pi_{sd}^{B}(z)$

If the firm sells to both types, it solves

$$\pi_{sd}^{B}(z) = \max_{p_{sd}^{H}(z), p_{sd}^{L}(z), \ell_{sd}(z)} p_{sd}^{H}(z)x_{sd}^{H}(z) + p_{sd}^{L}(z)x_{sd}^{L}(z) - w_s \ell_{sd}(z) \tag{11}$$

$$\text{s.t.} \quad p_{sd}^{H}(z) \leq (1+r)p_{sd}^{L}(z) \qquad \text{(the resale constraint)} \tag{12}$$

Within the borders of a given country, firms may be constrained by consumers buying the good at the low price and reselling it at a mark-up, limiting a firm's ability to price discriminate between consumers. This is represented by the resale constraint in equation (12). Specifically, if a firm makes its good available at a different price for each income group, r$\in [0, \infty]$ is the maximum sustainable difference in price that a firm can charge within the borders of a country before consumers engage in reselling. If $r = 0$, firms must charge the same price to both high and low income consumers, while if r is large enough the firm can perfectly price discriminate between the two income groups.

Selling to both consumer types if not resale constrained, $p_{sd}^{H} \leq (1+r)p_{sd}^{L}$

If the firm is not resale constrained, the assumptions of a linear labor-only production function and no exporting cost allows the problem of each income group in destination d to be solved separately and independently,

$$\pi_{sd}^{B;U}(z) = \pi_{sd}^{H}(z) + \pi_{sd}^{L}(z) \tag{13}$$

Specifically, the price charged to the k-type is given by equation (8) and the productivity cutoff by equation (9).

Using the optimal prices in an unconstrained economy, the necessary and sufficient condition for a non-binding resale constraint, $p_{sd}^{H} < (1+r)p_{sd}^{L}$, is

$$Q_{d}^{H} \geq (1+r)^2 Q_{d}^{L} \tag{14}$$

Therefore the constraint will bind (or not bind) for all firms serving the destination, independent of firm or source country characteristics.

Given Lemma 2.1, the productivity at which the firms begin selling to both consumers in the unconstrained economy, $z_{sd}^{B,U}$, is the productivity at which they begin selling to the L-type, z_{sd}^{L}.

Selling to both consumer groups if resale constrained

If the resale constraint binds, no firm selling in destination d can perfectly price discriminate, and prices must be solved jointly. Denote by p_{sd}^{B} the price charged to the L-type, so that $(1+r)p_{sd}^{B}$ denotes the price charged to the H-type. The firms problem in equation 11 becomes

$$\pi_{sd}^{B,C}(z) = \max_{p_{sd}^{B}(z),\ell_{sd}(z)} p_{sd}^{B}(z)\left(x_{sd}^{H}(z)(1+r)+x_{sd}^{L}(z)\right) - w_s\ell_{sd}(z) \tag{15}$$

The optimal price for a resale constrained firm selling to both types of consumer is

$$p_{sd}^{B}(z) = \underbrace{\left[\frac{\tau_{sd}w_s}{z}\right]^{\frac{1}{2}}}_{\text{Marginal Cost}} \underbrace{\left[\frac{M_d^H Q_d^H + (1+r)M_d^L Q_d^L}{(1+r)M_d}\right]^{\frac{1}{2}}}_{\text{Mark-up}} \tag{16}$$

where the mark-up is the population weighted sum of independent cutoff prices, adjusted by the resale constraint. As in the unconstrained economy, all firms selling to a consumer group-k in destination d charge the same mark-up.

The tension of the resale constrained is reflected in equation (16). Consider an increase in inequality created by changes in relative wages so that aggregate income, $Y_d = M_d^H w_d^H + M_d^L w_d^L$, remains constant. Higher H-type wages increases Q_d^H, pushing firm z to increase the price they charge when selling to both consumers. However, simultaneously Q_d^L decreases, reflecting the lower wages of the L-type consumers, pulling firm z to reduce the price charged to both consumers. The elasticity of Q_d^k with respect to changes in income is the determining factor in which force dominates. As I will show in subsequent sections, while income inequality will affect the economy the direction of that effect depends on the country's parameters and will not necessarily yield the same sign in all situations.

Lemma 2.2. *All else equal, if $Q_d^H > Q_d^L$ and the resale constraint is binding, then*

$$p_{sd}^H(z) > (1+r)p_{sd}^B \geq p_{sd}^B(z) > p_{sd}^L(z)$$

Proof: The resale constraint is only binding if $Q_d^H > (1+r)^2 Q_d^L$. The remainder of the proof follows algebraically when combined with equations 8, 16, and $r \geq 0$. □

Lemma 2.2 Firms in a resale constrained economy therefore face a choice: they can either set a high price, p_d^H and sell to subset of consumers, M_d^H; or they can set a low price and sell to the entire population.

Lemma 2.3. *In the constrained economy, if $Q_d^H > Q_d^L$ then firms initially only sell to the H-type consumers; $\pi^H(z_{sd}^L) \geq \pi_{sd}^{B,C}(z_{sd}^L)$*

Proof: Follows from $\pi_{sd}^H(z_{sd}^L) = \pi_{sd}^H(z_{sd}^L) + \pi_{sd}^L(z_{sd}^L) = \pi_{sd}^{B,U}(z_{sd}^L) \geq \pi_{sd}^{B,C}(z_{sd}^L)$, where the equality comes from the definition of $\pi_{sd}^L(z_{sd}^L) = 0$ and the inequality from unconstrained optimality of $\pi_{sd}^{B,U}$. □

Lemma 2.3 establishes that even in the constrained economy, firms will initially sell only the H-type consumers. The transition from shut-down to H-type sales, z_{sd}^H, takes the same form as equation 9. Under Lemma 2.1, firms will never sell to only the L-type group, so the only relevant transition is from H-type to selling to both types. This transition productivity can be found by solving $\pi_{sd}^H(z_{sd}^C) = \pi_{sd}^{B,C}(z_{sd}^C)$ to find

$$z_{sd}^C = \frac{w_s \tau_{sd}}{M_d^H Q_d^H} \left(\frac{M_d^L}{\psi_d} \right)^2 \tag{17}$$

where ψ_d contains only destination specific variables

$$\psi_d = \left(\frac{1}{1+r} + \zeta_d\right)^{\frac{1}{2}} (M_d + rM_d^H)^{\frac{1}{2}} - M_d^{H\frac{1}{2}} + \sqrt{\left[\left(\frac{1}{1+r} + \zeta_d\right)^{\frac{1}{2}} (M_d + rM_d^H)^{\frac{1}{2}} - M_d^{H\frac{1}{2}}\right]^2 - M_d^L \zeta_d} \tag{18}$$

and $\zeta_d = \frac{M_d^L Q_d^L}{M_d^H Q_d^H}$ is also introduced for notational convenience.

2.3. General Equilibrium of the World Economy

By Lemma 2.1 and 2.3, the productivity cutoffs are ordered as represented by Figure 1: Firms initially sell only to H-type, $\pi_{sd}^H(z)$, before permanently transitioning and selling to both types of consumers earning $\pi_{sd}^B(z)$. For clarity, z_{sd}^B will refer to the transition productivity from selling to only H-type to selling to both types, with z_{sd}^L and z_{sd}^C denoting the transition productivity for the unconstrained and constrained economies respectively.

(Figure 1 about here)

Given the higher productivity requirements of L-type consumers, the firms they purchase from are more likely to be exporters. This result is consistent with the empirical observations of Fajgelbaum and Khandelwal (2016), who find that the poor spend more on the most traded sectors (in this model, the more productivity firms), while high income consumers spend more on the least traded sectors (the lower productivity firms).

I use the Chaney (2008) simplification and assume that firms can pay a fixed start-up cost, f_c, to draw a productivity z from a Pareto distribution with support $[b_s, \infty]$ where b_s denotes the minimum productivity level of the draw in country s.[4] The number of firms in s who have paid the fixed cost to draw a productivity is denoted by J_s: this represents the number of potential firms who could produce goods in the economy if their productivity is high enough. Combining the expressions of trade value with the assumptions of balanced

[4]The Pareto cdf is $F(z) = 1 - \left(\frac{b_s}{z}\right)^{\gamma}$, the pdf is $f(z) = \frac{\gamma b_s^{\gamma}}{z^{\gamma+1}}$, where the shape parameter $\gamma > 0$.

trade, labor market clearing, law of large numbers, and zero profit conditions[5], the number of potential firms in each country is

$$J_s = \frac{L_s}{\left[(\gamma+1) f_c + \underbrace{\frac{\gamma q r}{w_s} \sum_{d,C} \left(\frac{b_s}{z_{sd}^{B,C}} \right)^{\gamma} \left(\frac{M_d^H Q_d^{H,C}}{1+r} - \frac{M_d^H}{M_d^L} \left(M_d^H Q_d^{H,C} + M_d^L Q_d^{L,C} \right) \right)}_{\text{Global Effect of Inequality}} \right]} \tag{19}$$

where x^C denotes a variable in a resale constrained economy. The summation term over resale constrained economies in the denominator is unique to this model: the number of firms react to the difference in consumer cut-offs - but only if there are economies bound by the resale constraint, and if imperfect price discrimination is allowed, $r > 0$.

Combining equation 19, balanced trade, exploiting $z_{sd}^k = \frac{w_s \tau_{sd}}{w_d} z_{dd}^k$, and recalling that the resale constraint either binds or does not bind for all firms exporting to d (so all firms earn R_{sd}^U or R_{sd}^C), yields the wage formula that mimics the standard functional form for this class of model

$$w_d^{\gamma} = \sum_t \frac{Y_t}{Y_d} \frac{J_d \left(\frac{b_d}{\tau_{dt}} \right)^{\gamma}}{\sum_s J_s \left(\frac{b_s}{\tau_{st} w_s} \right)^{\gamma}} \tag{20}$$

such that wages in d is a function of wages, GDP, and potential firms in all other countries. Additionally, if there are resale constrained economies with imperfect price discrimination, wages will, through the potential number of firms, equation 19, incorporate inequality information in the general equilibrium, which differentiates this model from the standard results.[6]

Definition 2.4. *Given trade barriers τ_{sd}, a fixed cost of entry, f_c, a productivity distribution F(z) with a lower bound b_s, and income distribution information M_d^k and θ_d^k; an equilibrium*

[5]Details of all calculations can be found in the online appendix.

[6]Unlike traditional models, w_d is not GDP per capita, it is GDP per effective labor unit. To see this, note that GDP$=M_d^H w_d^H + M_d^L w_d^L = w_d \left(M_d^H \theta_d^H + M_d^L \theta_d^L \right)$. Therefore, $\frac{GDP}{M_d} = w_d \frac{M_d^H \theta_d^H + M_d^L \theta_d^L}{M_d} \neq w_d$.

for s,d=1,..I and $\forall z \in [b_s, \infty)$ *is given by: a productivity threshold for each consumer type k,* $\hat{z}^k_{sd}$*; a measure of entrants in each country,* $\hat{J}_s$*, the total measure of firms serving group k market d,* $\hat{N}^k_d$*, a aggregate price statistic for each k-type,* $\hat{P}^k_d$*, wages for each type,* $\hat{w}^k_d$*; per consumer consumption,* $\hat{c}^k_{sd}(z)$*; a demand function for each firm* $\hat{x}^k_{sd}(z)$ *and firm pricing rules* $\hat{p}^k_{sd}(z)$ *such that (i) Given* $\hat{P}^k_d$*,* $\hat{N}^k_d$*,* $\hat{w}^k_d$*, and* $\hat{p}_{sd}(z)$*, the representative consumer solves her maximization problem,(1), by choosing* $\hat{c}^k_{sd}(z)$ *according to (2) (ii) Given the demand function* $\hat{x}^k_{sd}$*, firm z solves its maximization problem (6) by choosing prices* $\hat{p}^k_{sd}(z)$ *according to (8) or (16) (iii) The productivity threshold for* s *selling in* d *satisfies (9) or (17). (iv) Goods market clear:* $\hat{x}^k_{sd}(s) = M^k c^k_{sd}(z)$ *(v) Trade is balanced:* $\sum_{d \neq s} R_{sd} = \sum_{s \neq d} \sum_k p^k_{sd} x^k_{sd}$ *(vi) Labor markets clear:* $\sum_d \int_z \ell_{sd}(z) dz = M^H_s \theta^H_s + M^L_s \theta^L_s$*. (vii)* N^k_d*,* P^k_d*,* J_s*, and* w^k_d*, jointly satisfy (4), (5), (19), and (20).*

3. Trade Effects of Income Inequality

3.1. Unconstrained economy

The unconstrained economy correctly implies that variety and firm prices will increase, but incorrectly suggests no relationship between trade revenue, and a postive relationship between unit value and inequality.

Trade Revenue

Trade revenue for an unconstrained economy is

$$R^U_{sd} = \underbrace{\frac{J_s b_s^{\gamma} (w_s \tau_{sd})^{-\gamma}}{\sum_t J_t b_t^{\gamma} (w_t \tau_{td})^{-\gamma}}}_{\text{Indirect}} \underbrace{Y_d}_{\text{Direct}} \tag{21}$$

Any change in inequality that does not change aggregate income will not directly impact trade revenue. The indirect effect of inequality is introduced through the dependence of number of firms on inequality in equation 19. Economies that are not resale constrained are not incorporated in the denominator of equation 19, and therefore changes in inequality in unconstrained economies will not impact the general equilibrium variables, however, the indirect channel allows for inequality changes in constrained economies to impact unconstrained economies.

Changing inequality in an unconstrained economy does not impact trade revenue due to the identical, constant income elasticity: the share of spending for each consumer type is linearly proportional to their income,

$$R_{sd}^{k,U} = M_d^k \int p_{sd}^k(z) c_{sd}^k(z) = \frac{J_s b_s^{\gamma} (w_s \tau_{sd})^{-\gamma}}{\sum_s J_s b_s^{\gamma} (w_s \tau_{sd})^{-\gamma}} M_d^k w_d^k \tag{22}$$

So that the share of purchases of type k in aggregate trade is their share of aggregate income

$$TS_{sd}^k = \frac{R_{sd}^{k,U}}{R_{sd}^U} = \frac{M_d^k w_d^k}{Y_d} \tag{23}$$

An increase in inequality would therefore reallocate traded goods across the two income groups, increasing the share purchases by the H-type and decreasing the share of L-type, but these changes would exactly off-set each other.

Variety

The variety of imported goods is determined by z_{sd}^H, which is determined by only the H-type consumer. Combining equation 9 and 39 yields

$$z_{sd}^{H,U} = \frac{\tau_{sd} w_s}{\left[\frac{(1+2\gamma) w_d}{q \sum_s J_s b_s^{\gamma} (w_s \tau_{sd})^{-\gamma}}\right]^{\frac{1}{1+\gamma}} \theta_d^{H \, \frac{1}{1+\gamma}}} \tag{24}$$

This productivity cutoff decreases as θ_d^H increases, indicating that more income unequal economies import a larger variety of differentiated goods.

Unit Value

Aggregate quantity purchased by the economy is

$$C_{sd}^{U} = \gamma b_s^{\gamma} J_s \left[\int_{z_{sd}^{H}}^{\infty} M_d^H c_d^H(z) z^{-(\gamma+1)} dz + \int_{z_{sd}^{L}}^{\infty} M_d^L c_d^L z^{-(\gamma+1)} dz \right]$$
$$= \underbrace{\frac{q}{2(\gamma-\frac{1}{2})} \left(\frac{(1+2\gamma) w_d}{q} \right)^{\frac{\gamma}{1+\gamma}} \frac{J_s b_s^{\gamma} (\tau_{sd} w_s)^{-\gamma}}{\left(\sum_s J_s b_s^{\gamma} (w_s \tau_{sd})^{-\gamma}\right)^{\frac{\gamma}{1+\gamma}}}}_{A_d} \left[M_d^H \theta_d^{H^{\frac{\gamma}{1+\gamma}}} + M_d^L \theta_d^{L^{\frac{\gamma}{1+\gamma}}} \right] \quad (25)$$

As this economy is not resale constrained, a change in inequality does not have an indirect impact on the number of firms or wages, and therefore the term A_d remains constant.

$$\frac{\partial C_{sd}^{U}}{\partial \theta_d^H} = \frac{\gamma A_d M_d^H}{1+\gamma} \left[\theta_d^{H^{\frac{-1}{1+\gamma}}} - \theta_d^{L^{\frac{-1}{1+\gamma}}} \right] < 0 \qquad \text{if } \theta_d^H > \theta_d^L \quad (26)$$

Quantity therefore decreases with inequality. Unit value is the aggregated revenue divided by the aggregated quantity, $UV_{sd} = \frac{R_{sd}^U}{C_{sd}^U}$, yielding

$$UV_{sd}^{U} = \frac{R_{sd}^{U}}{C_{sd}^{U}} = \frac{\gamma - \frac{1}{2}}{\gamma + \frac{1}{2}} \frac{M_d^H Q_d^{H,U^{1+\gamma}} + M_d^L Q_d^{L,U^{1+\gamma}}}{M_d^H Q_d^{H,U^{\gamma}} + M_d^L Q_d^{L,U^{\gamma}}} \quad (27)$$

As revenue is not affected by a change in inequality, but quantity decreases, unit value increases.

3.2. Constrained economy

Variety

The number of varieties is still determined by only the H-type, and is still given by equation 39. As Q_d^H increases with H-income (an inequality increase), the imported variety increases.

Revenue

The choke prices in the constrained economy, equations 47 and 48, do not have constant income elasticity and so aggregates no longer have a constant income elasticity either. Calculating trade revenue yields a more complicated expression for the direct channel

$$R^C_{sd} = \underbrace{\frac{J_s b_s^{\gamma} (w_s \tau_{sd})^{-\gamma}}{\sum_t J_t b_t^{\gamma} (w_t \tau_{td})^{-\gamma}}}_{\text{Indirect}} \underbrace{\left[Y_d + \overbrace{\left(M_d + rM_d^H\right)^{\frac{1}{2}} \left(\left(M_d + rM_d^H\right)^{\frac{1}{2}} - M_d^{\frac{1}{2}} \right) P_d^L}^{\text{Positive if } r>0} \right]}_{\text{Direct}} \tag{28}$$

where

$$P_d^{L,R} = \frac{\overbrace{2\gamma w_d^L}^{\text{Decreases}}}{q} \underbrace{\left[\overbrace{\frac{2\left(\gamma + \frac{1}{2}\right) M_d^{\frac{1}{2}}}{\psi_d}}^{\text{Increases}} \overbrace{\frac{\zeta_d}{\left(\frac{1}{1+r} + \zeta_d\right)^{\frac{1}{2}}}}^{\text{Decreases}} - 2\gamma \right]^{-1}}_{\text{Impact of Resale Constraint}} \tag{29}$$

If $r = 0$, the second term is eliminated, and the revenue behaves in the same way as the constrained economy. In a resale constrained economy, the price and cutoff for the goods purchased by both types of consumers, p^B_{sd} and $z^{B,R}_{sd}$, incorporates information from both H income and L income changes through the cutoff prices. As such, as long as $r \neq 0$, trade revenue can increase or decrease as a result of a change in inequality. This is also true for trade quantity and unit value

Unit Value

The quantity of goods is given by

$$C_{sd}^{C} = q\left[\overbrace{\frac{\frac{1}{2}M_d^H N_{sd}^H}{\gamma-\frac{1}{2}} - M_d^L N_{sd}^L}^{\text{Increase}} + \frac{\gamma}{\gamma-\frac{1}{2}}\overbrace{\frac{M_d^L N_{sd}^L}{\psi_d}}^{\text{Depends}}\overbrace{\left(M_d^{\frac{1}{2}}\left(\frac{1}{1+r}+\zeta_d\right)^{\frac{1}{2}} - M_d^{H\frac{1}{2}}\right)}^{\text{Decrease}}\right] \tag{30}$$

Which, like trade revenue, has to potential to either increase or decrease.

Defining unit value as before, $UV_{sd}^{R} = \frac{R_{sd}^{R}}{C_{sd}^{R}}$ yields

$$UV_{sd}^{C} = M_d^H Q_d^H \frac{M_d^{H^{-\gamma}}\left(\frac{M_d^L}{\psi_d}\right)^{2\gamma} + (1+2\gamma)\,\zeta_d + \frac{\gamma\psi_d}{M_d^L}\left[M_d^{H^{\frac{1}{2}}} - \left(M_d + rM_d^H\right)^{\frac{1}{2}}\left(\frac{1}{1+r}+\zeta\right)^{\frac{1}{2}}\right]}{2\left(\gamma+\frac{1}{2}\right)\left[\frac{\frac{1}{2}M_d^{H^{1-\gamma}}}{\gamma-\frac{1}{2}}\left(\frac{M_d^L}{\psi_d}\right)^{2\gamma} - M_d^L + \frac{\gamma}{\gamma-\frac{1}{2}}\frac{M_d^L}{\psi_d}\left(M_d^{\frac{1}{2}}\left(\frac{1}{1+r}+\zeta_d\right)^{\frac{1}{2}} - M_d^{H^{\frac{1}{2}}}\right)\right]} \tag{31}$$

Which can either increase or decrease with inequality depending on parameter values.

3.3. At least one economy must be constrained

Equation 21 depends only on aggregate income, which contradicts empirical evidence that shows that trade value increases with inequality. Therefore at least economy must be constrained, imposing an upper bound on the value of r.

Using equation 14 and 39, it is possible to show that an economy is unconstrained if

$$(1+r)^2 > \left(\frac{w_d^H}{w_d^L}\right)^{\frac{1}{1+\gamma}} \tag{32}$$

Recall that the measure of inequality is the income share of the upper 40%, so $I_d^{40} = \frac{M_d^H w_d^H}{Y_d}$, where I_d^{40} is the value from the data. Whether an economy is unconstrained for a given value of r can be determined without solving for the general equilibrium by checking

$$(1+r)^2 > \left(\frac{M_d^L}{M_d^H}\frac{I_d^{40}}{1-I_d^{40}}\right)^{\frac{1}{1+\gamma}} \tag{33}$$

Using data and equation 33, the binding value of r can be found for each economy, reported in Table 1. The largest value of r is $r^{bind} = 21.63\%$, found for the United States. If $r > 0.2163$, firms in all countries can freely price discriminate, which would contradict empirical evidence on trade flows. Therefore, it must be that $r \leq 0.2163$.

Unlike an unconstrained economy, the indirect channel will transmit changes in the resale constrained economy to all other economies as the number of firms adjust as long as $r \neq 0$ changes in inequality may impact trade for the resale constrained economy. Similarly, the productivity impact from the direct channel only appears if $r \neq 0$. This imposes a lower bound on r, so that it must be that $r \in (0, 0.2163)$ for inequality to impact trade.

4. Calibration and Simulation: Trade and Inequality

4.1. Calibration

(Table 1 about here)

This model contains 6 country-specific parameters: M_d, M_d^H,M_d^L, b_d, θ_d^H, θ_d^L, 1 bilateral parameter: τ_{sd}, and 5 country-independent parameters: N, γ, f_c, q, and r.

4.1.1. Non-Specific Parameters: N, γ, b_d, q, f_c

I set N equal to match the number of countries in my sample plus the USA (55 countries), and use $\gamma = 2.8$ from Simonovska and Waugh (2014). Under the assumptions that b_d, f_c, and q are the same across all countries, and the linearity of log utility and production function, the regression results of trade value and inequality are independent of these three parameters. Therefore, f_c and q are normalized to 1, while the lower bound on the productivity draw is set to be less than the minimum productivity cutoff for all countries. As b_d is the same across countries, any cross country differences in productivity will be induced by differences in labor productivity, θ_d^H or θ_d^L.

4.1.2. Country Specific Parameters: $M_d, M_d^H, M_d^L, \theta_d^H, \theta_d^L$

I set M_d equal to population reported by WDI, and in keeping with my metric for inequality, choose M_d^H to be 40% of the population, which then defines M_d^L to be 60% of the population. Rearrange the definition of income share and GDP to find that $\theta_d^H = \frac{Y_d IR_d}{w_d M_d^H}$ and $\theta_d^L = \frac{1-IR_d}{IR_d}\frac{M_d^H \theta_d^H}{M_d^L}$. I combine these equations with (20) to find parameter values for θ_d^H, θ_d^L to simultaneously match GDP and the income shares given the model calculated wages. As income share information for the USA is not available from the WDI, I use USA income share information obtained from the U.S. Census Bureau, table H-2.

4.1.3. Bilateral Parameter, τ_{sd}

To estimate trade barriers I follow the gravity equation methodology of Eaton and Kortum (2002) (EK). The expression for import share of s goods in d expenditures is

$$\lambda_{td} = \frac{R_{td}}{\sum_s R_{sd}} = \frac{b_t^\gamma J_t \left(w_t \tau_{td}\right)^{-\gamma}}{\sum_s b_s^\gamma J_s \left(w_s \tau_{sd}\right)^{-\gamma}} \tag{34}$$

in both types of economies (constrained or unconstrained). This can be manipulated to find that the relative share of goods imported to goods produced for domestic consumption is given by

$$\log\left(\frac{\lambda_{sd}}{\lambda_{dd}}\right) = \log\left(b_s^\gamma J_s \left(w_s \tau_{sd}\right)^{-\gamma}\right) - \log\left(b_d^\gamma J_d \left(w_d \tau_{dd}\right)^{-\gamma}\right) \tag{35}$$

Define B_s and B_d as exporter- and importer-fixed effects to correspond to $\log\left(b_s^\gamma J_s w_s^{-\gamma}\right)$ and $\log\left(b_d^\gamma J_d w_d^{-\gamma}\right)$ respectively, and use $\tau_{dd} = 1$ to obtain the standard EK gravity equation

$$\log\left(\frac{\lambda_{sd}}{\lambda_{dd}}\right) = B_s - B_d - \gamma \log\left(\tau_{sd}\right) \tag{36}$$

where I assume the following functional form for trade barriers:

$$\log(\tau_{sd}) = log(d_{sd}) + b_{sd} + \delta_{sd} \tag{37}$$

he distance between s and d is captured by d_{sd}, b_{sd} is an indicator variable that takes a value of 1 if s and d share a border, and δ_{sd} represents the error.

For information on distance and shared borders I use CEPII estimates. To construct trade shares for λ_{sd} I use COMTRADE data on the sum of reported exports between sample countries in BEC 522, 61, 62 and 63. If exports data does not exist, I use reported imports instead. Traditionally, λ_{dd} is calculated as the residual of GDP that is not exported, and for this purpose of this paper should correspond to the production in categories BEC 522, 61, 62 and 63. As this data does not exist, I instead use the doubled value added in manufacturing (ISIC 15-37) obtained from the World Bank. This allows me to apply least squares to estimate the trade barriers for all 55 countries that are consistent with the observed trade flows..

4.1.4. Resale Constraint, r

I simulate the model over a grid of r values to find the level of price discrimination that minimizes the distance between the simulated and measured moments. I collect the 2011 trade value, TV_{sd}, and trade quantity, TQ_{sd}, for goods at the 6-digit Harmonized System level (HS6) exported from the USA to all trading partners from COMTRADE (2014) (Commodities Trade Statistics database). I discard observations that report a unit value of less than 1% or more than 100 times the median unit value of that HS product line, where unit value, UV_{sd} is generated by dividing trade flow value by kilogram quantity of goods traded, $UV_{sd} = TV_{sd}/TQ_{sd}$. As my regression exploits differences in income inequality across destinations, I drop any product line the USA exports to fewer than five destinations.

I collect 2011 data on population, GDP per capita, and income inequality between 2009-2013 from the WDI (2014) (World Development Indicators). If it exists, inequality data from

2011 is used, if it does not the year closest to 2011 is used, with the average value if two observation are equidistant. For ease of mapping to the model, my measure of inequality is the income share of the upper 40%. As this number increases, more of the aggregate income is held by the upper 40%, representing increasing inequality.

Equation (38) summarizes my regression framework; x_d^V is the trade value, β^{ci} is the elasticity of x_d^i with respect to income per capita, β^{mi} is the elasticity with respect to market size (as proxied by population), and β^{si} is elasticity with respect to inequality, measured by the share of income held by the upper 40% of income earners.

$$log(x_d^i) = \alpha + \beta^{ci}\log(\text{CGDP}_d) + \beta^{mi}\log(\text{Pop}_d) + \beta^{si}\log(\text{Inequality}_d) + \varepsilon_{di} + controls \quad (38)$$

I use additional controls for (the log of) distance and an indicator for a common official language obtained from the CEPII GeoDist database (Mayer and Zignago (2011)), an indicator for 2011 EU, NAFTA, ASEAN, or MERCOSUR membership, HS-fixed effects to control for product specific effects, and cluster errors by destination.

The model employed in this paper examines the impact of income distribution on trade of differentiated consumption goods, therefore I use the 2007 revision of Rauch (1999) production differentiation index to identify the differentiated goods by HS code, and World Bank WITS (2014) concordance to map the HS6 categories into the BEC (Broad Economic Classification) categories to identify consumption goods.[7]

(Table (2) about here)

Table (2) summarizes the regression results of equation 38 for trade value, quantity, and unit value for differentiated consumption goods. Income inequality increases trade value and quantity, but decreases unit value, and has the greatest impact on the effect of average income.

[7]See unstats.un.org/unsd/publication/SeriesM/SeriesM_53rev4e.pdf for further details about BEC classification.

(Table (3) about here)

Food production subsidies, trade tariffs, and aid flows can vary greatly for non-market reasons. Table 3 repeats the exercise of Table (2) but removes all HS products lines associated with food. The effect of inequality increases when food is removed from differentiated consumer goods trade. A 1% increase in inequality corresponds with a 9% increase in trade value, a 10% increase in quantity, and a 1% reduction in unit values. These results are similar to those found by Bekkers et al. (2012) who study the effects of inequality on unit values, taking care to only use consumption goods, and finds similar results to those presented using the more standard Atkinson index instead of the income share.

4.2. Simulation Results

Figure 2 presents the regression coefficient on inequality from equation 38 regressed on simulated data. Changing price discrimination has only a minimal impact on the relationship between import value and inequality, generates a mostly positively-valued inverted-u-shape relationship for quantity, and a mostly negatively-valued u-shape with unit value. The "steps" are discontinuous jumps in the regression coefficient that occur as countries switch from binding to non-binding constraints.

(Figure 2 about here)

Table (4) contains the results for selected values of r: an global economy that does not allow price discrimination (r=0%), an globally unconstrained economy (r=25%), and values of r that generate results that are the closest distance to the elasticity found for GDP per capita, population, and inequality respectively. In all cases except for the unconstrained unit value results, the model generates response in the correct direction, so that changing the degree of price discrimination impacts only the magnitude of the response.

(Table 4 about here)

The trade value coefficients for GDP per capita and population do not change with r. The difference between the coefficient of trade value and inequality obtained from the data and from the simulated data is minimized at r=11.7%. The model with this value of r can generate 53% of the variations of trade with inequality.

Whether targeting GDP per capita or population, the r value that minimizes the distance for trade quantity coefficients is $r = 10.0\%$, while the optimal r for Inequality is $r = 15.8\%$, which overestimates the effect of inequality on quantity by one percentage point, predicting an 11% increase instead of a 10% increase.

Finally, the model generates a negative relationship between inequality and unit values. GDP per capita is best duplicated under $r = 0\%$. As population does not have a statistically significant relationship, a value of r cannot be targeted, though any value of $r \in [11.0\%, 16.8\%]$ generates results with no statisical significant relationship between population and unit value in the simulated data. Finally, the r value that best duplicates the relationship between inequality and unit value is $r = 5.3\%$, but again overestimates the impact of inequality, by one percentage point.

Few micro-studies exists that estimate within-country price discrimination by income group, and most studies that do exist have information on prices at different locations but not the goods purchased by the different groups. One of the few studies that has both price and purchasing data is Broda et al. (2009). Using scanner data from USA households in 2005 they find that high income households pay 5.1% more than low income households for goods of similar quality, close to the value prescribed by the model when using trade unit value data.

5. Firm-Level Effects of Income Inequality

In this section, I define a change in inequality to be a change in relative productivities that does change the effective aggregate effective labor supply, L_d. This is accomplished by changing θ_d^H and defining $\theta_d^L = \frac{L_d - M_d^H \theta_d^H}{M_d^L}$.

5.1. An Unconstrained Economy

Equilibrium Cutoff Prices, Q_d^k

Using model solutions, the cutoff price in an unconstrained economy can be expressed as

$$Q_d^{k,U} = \underbrace{\left[\frac{w_d}{q\sum_s J_s b_s^{\gamma}(w_s\tau_{sd})^{-\gamma}}\right]^{\frac{1}{1+\gamma}}}_{\text{General Equilibrium Term}} \underbrace{\left[(1+2\gamma)\theta_d^k\right]^{\frac{1}{1+\gamma}}}_{\text{k-type term}} \tag{39}$$

General equilibrium effects have the same impact on both consumers-types cutoff prices within an unconstrained economy — the difference in cutoff prices comes stems from differing productivities.

Proposition 5.1. *If θ_d^k increases then $Q_d^{k,U}$ increases.*

Proposition 5.2. *If θ_d^H increases and θ_d^L decreases such that $L_d = M_d^H\theta_d^H + M_d^L\theta_d^L$ remains constant, then $Q_d^{H,U}$ increases and $Q^{L,U}$ decreases.*

Proof of Propositions 5.1 and 5.2 follow from equations (19), (20), and (39).

This can be used in equation (14) to yield that an economy is not resale constrained if

$$(1+r)^2 > \left(\frac{w_d^H}{w_d^L}\right)^{\frac{1}{1+\gamma}} \tag{40}$$

Firm Level Effects Within an Unconstrained Economy

An increase in inequality will always increase the price and profit of a firm that sells to only the H-type of consumer. A more interesting case is that of the firm that sells to both types. The relative price within an economy of a good sold to both groups by the same firm is given by

$$\frac{p_{sd}^{H,U}(z)}{p_{sd}^{L,U}(z)} = \left(\frac{\theta_d^H}{\theta_d^L}\right)^{\frac{1}{2(1+\gamma)}} \tag{41}$$

An increase in income inequality within a country will increase the difference in relative mark-ups. However as $\frac{1}{2(1+\gamma)} < 1$ relative prices will change by less than than income inequality.

The final group of goods to consider is those that switch from being sold to both types to being sold to one type. Fortunately, an increase in inequality means that for any given firm, the transition will be from selling to both types to only one type, therefore resulting in an increase in price.

The relative profit a firm earns from from each group is given by

$$\frac{\pi_d^H(z)}{\pi_d^L(z)} = \underbrace{\frac{M_d^H}{M_d^L}}_{<1} \underbrace{\left(\frac{{Q_d^H}^{\frac{1}{2}} - \left(\frac{\tau_{sd} w_s}{z}\right)^{\frac{1}{2}}}{{Q_d^L}^{\frac{1}{2}} - \left(\frac{\tau_{sd} w_s}{z}\right)^{\frac{1}{2}}} \right)}_{>1} \tag{42}$$

The majority source of profit for a firm depends on the relative productivity of the two types, their population share, and the firms own productivity. For a given economy, it is possible for some, low z firms to receive the majority of their revenue from H-type consumers, while others with higher productivity receive the majority of their revenue from L-type consumers. Similarly, some firms could see an increase in revenue if inequality rises, while others a decrease.

Firm Level Effects Across Unconstrained Economies

The mark-up component in equation 8 determines whether prices in more unequal countries will be higher or lower. The price of goods sold only to the H-type of consumers in both countries increases as inequality increases.

$$\frac{p_{sd}^H(z)}{p_{sj}^H(z)} = \left(\frac{\tau_{sd}}{\tau_{sj}}\right)^{\frac{1}{2}} \left(\frac{Q_d^H}{Q_j^H}\right)^{\frac{1}{2}} \tag{43}$$

$$= \left(\frac{\tau_{sd}}{\tau_{sj}}\right)^{\frac{1}{2}} \left(\frac{\sum_s J_s b_s^{\gamma} \left(w_s \tau_{sj}\right)^{-\gamma}}{\sum_s J_s b_s^{\gamma} \left(w_s \tau_{sd}\right)^{-\gamma}}\right)^{\frac{1}{2(1+\gamma)}} \left(\frac{w_d^H}{w_j^H}\right)^{\frac{1}{2(1+\gamma)}} \tag{44}$$

The case of a firm that sells to both consumers is more complicated as data sets for firms report only one price for identical goods, even though the good may have multiple prices in the destination market. If one assumes that, in the destination market, the L-type price is marketed as a discount on the full price, the equation above applies and higher inequality countries report higher prices of imported goods.

Alternatively, the empirical price measure can be constructed from a good level unit-value, dividing the revenue by the quantity sold. For a firm selling to both consumers this good level unit-value is

$$p_{sd}^{UV,U}(z) = \frac{R_{sd}^{H,U}(z) + R_{sd}^{L,U}(z)}{M_d^H c_d^H(z) + M_d^L c_d^L(z)} \tag{45}$$

$$= \frac{M_d^H Q_d^{H\frac{1}{2}} + M_d^L Q_d^{L\frac{1}{2}} - M_d \left(\frac{\tau_{sd} w_s}{z}\right)^{\frac{1}{2}}}{M_d^H Q_d^H + M_d^L Q_d^L - \left(M_d^H Q_d^{H\frac{1}{2}} + M_d^L Q_d^{L\frac{1}{2}}\right)\left(\frac{\tau_{sd} w_s}{z}\right)^{\frac{1}{2}}} \tag{46}$$

As was the case with profit, the model contains tensions in both directions: the higher income of the H-type increases the unit-value price, while the lower income of the L-type decreases it. Whether the relative price increases or decreases across countries depends on relative productivities, population shares, and firm productivity.

5.2. Constrained economy

Equilibrium Cutoff Prices, Q_d^k

The constrained economy cutoff can be expressed as

$$Q_d^{H,C} = \left[\frac{w_d}{q \sum_s J_s b_s^{\gamma} (\tau_{sd} w_s)^{-\gamma}}\right]^{\frac{1}{1+\gamma}} \left[(1+2\gamma)\,\theta_d^H + 2\gamma\theta_d^L \psi_d \frac{(1+r)\left(\frac{1}{1+r} + \zeta_d\right)^{\frac{1}{2}} - \left(\frac{M_d}{M_d^H}\right)^{\frac{1}{2}}}{\zeta_d M_d^{\frac{1}{2}} - \frac{\gamma}{\gamma+\frac{1}{2}} \psi_d \left(\frac{1}{1+r} + \zeta_d\right)^{\frac{1}{2}}}\right]^{\frac{1}{1+\gamma}} \tag{47}$$

$$Q_d^{L,C} = \left[\frac{w_d}{q \sum_s J_s b_s^{\gamma} w_s \tau_{sd}^{-\gamma}}\right] \frac{\theta_d^L M_d^{\frac{1}{2}} \zeta_d}{\left[\zeta_d M_d^{\frac{1}{2}} - \frac{\gamma}{(\gamma+\frac{1}{2})} \psi_d \left(\frac{1}{1+r} + \zeta_d\right)^{\frac{1}{2}}\right]} \left(\frac{\psi_d}{M_d^L}\right)^{-2\gamma} \left(M_d^H Q_d^{H,R}\right)^{-\gamma} \tag{48}$$

While the cutoff prices in the unconstrained economy only incorporated information from the $k - type$, in a constrained economy cutoff prices reflect the inequality of a country by incorporating information on both productivities.

The new term in the H-type cutoff reflect the benefit of lower prices, from Lemma 2.2, that the H-type accrues due to the constrained pricing on the goods purchased by both consumer types, making the H-type amenable to paying higher prices on the exclusive goods. Conversly, the L-type consumer now pays a higher price on all goods therefore there cutoff price decreases.

Firm Level Effects Within a Constrained Economy

Within an economy, prices charged on goods sold to both groups differ by exactly $(1+r)$ by construction.

Examining the ratio of profit from each group yields

$$\frac{\pi_d^L(z)}{\pi_{sd}^H(z)} = \underbrace{\frac{M_d^L}{M_d^H}}_{>1} \underbrace{\frac{\left(Q_d^{L,C}\left[\frac{1}{S_d^r}\right]^{\frac{1}{2}} - \left[\frac{\tau_{sd}w_s}{z}\right]^{\frac{1}{2}}\right)}{\left(\frac{Q_d^{H,C}}{1+r}\left[\frac{1}{S_d^r}\right]^{\frac{1}{2}} - \left[\frac{\tau_{sd}w_s}{z}\right]^{\frac{1}{2}}\right)}}_{\text{Usually } <1} \underbrace{\frac{\left(S_d^{r\frac{1}{2}} - \left[\frac{\tau_{sd}w_s}{z}\right]^{\frac{1}{2}}\right)}{\left((1+r)S_d^{r\frac{1}{2}} - \left[\frac{\tau_{sd}w_s}{z}\right]^{\frac{1}{2}}\right)}}_{<1} \tag{49}$$

where $S_d^r = \frac{M_d^H Q_d^{H,C}}{1+r} + M_d^L Q_d^{L,C} = M_d^H Q_d^{H,C}\left(\frac{1}{1+r} + \zeta_d\right)$. Equation 49 contains the same elements as equation 42 —though in a more complex way due to the co-dependence of cutoff prices — the dominant group in a firm's profit depends on the relative income, group size, firm productivity, and the size of the arbitrage band.

Firm Level Effects Across Constrained Economies

If the firm sells the good to H-types in both economics, the relative prices are

$$\frac{p_{sd}^H(z)}{p_{sj}^H(z)} = \left(\frac{\tau_{sd}}{\tau_{sj}}\right)^{\frac{1}{2}} \left(\frac{Q_d^{H,C}}{Q_j^{H,C}}\right)^{\frac{1}{2}} \tag{50}$$

Again, this is similar to the unconstrained case, but now details of inequality are embedded within the cutoff prices. However, given that Q_d^H increases with p_d^H, equations 8 and 16 show that the goods that are sold to only the H-type have a mark-up increase. It is, however, unclear what happens to the price of a firm that continues to sell its goods to both types: these prices can either be driven to increase due to the increase in the cutoff price of the H-type, or it can decrease due to the lower income of the L-type. Therefore, unlike the behavior of the unconstrained economy, it is not clear that mark-ups will increase with inequality: it is even possible that they decrease.

If the price reported is

$$\frac{p_{sd}^B}{p_{sj}^B} = \left(\frac{\tau_{sd}}{\tau_{sj}}\right)^{\frac{1}{2}} \left(\frac{S_d^r}{S_j^r}\right)^{\frac{1}{2}} \tag{51}$$

which depends on parameters, and can capture a decreasing relationship between inequality and mark-ups.

Therefore, in both the constrained and unconstrained economies, mark-ups for individual goods can either or decrease as a result of an increase in inequality, consistent with the results of both Flach and Janeba (2017) (increase) and Simonovska (2015) (decrease).

6. Conclusion

In this paper I replicate the effects of importer inequality on imports with an income-based price discrimination model. My model can correctly generate the the negative relationship between inequality and unit value. I can replicate 50% of the empirical relationship between trade value and inequality in my dataset using the model, though I overestimate the relationship between inequality and unit value.

The model also generates the result, consistent with the observations of Fajgelbaum and Khandelwal (2016), that the poor spend will spend more in traded sectors (in this case, a firm), while high income consumers spend more on the least traded sectors. Finally, it generates some new predictions: (i) for countries facing sufficiently low wages and barriers, firms will produce and export without selling goods in the domestic market, (ii) The number of firms in a country depend on the inequality of neighboring countries.

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1. Appendix A: Sample Country Details

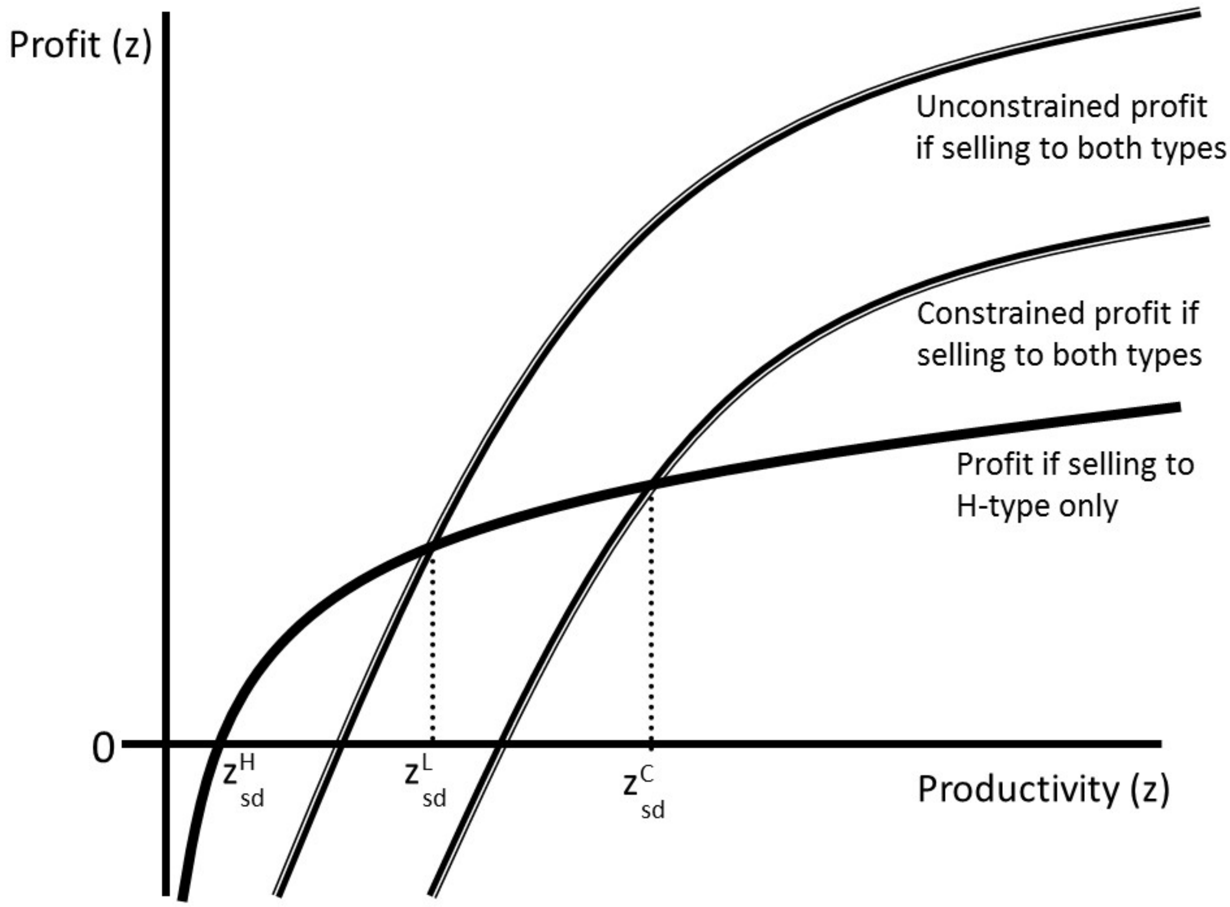
The following lists the income share of the upper 40 percentile (U40) in the aggregate income for each country in the sample, as well as the desired degree of price discrimination of firms, $r_d^{bind} = \left(\frac{Q_d^{H,N}}{Q_d^{L,N}}\right)^{\frac{1}{2}} - 1 = \left(\frac{\theta_d^H}{\theta_d^L}\right)^{\frac{1}{2(1+\gamma)}} - 1$. Using data on income share of the upper 40%, $IR_d = \frac{M_d^H w_d \theta_d^H}{Y_d}$ and $1 - IR_d = \frac{M_d^L w_d \theta_d^L}{Y_d}$, this simplifies to $r_d^{bind} = \left(\frac{3}{2}\frac{IR_d}{(1-IR_d)}\right)^{\frac{1}{2(1+\gamma)}} - 1$.

Table 1: Binding values of the resale constraint, r

Country	U40 Share (%)	r^{bind} (%)	Country	U40 Share (%)	r^{bind} (%)
Angola	58.27	10.22	Nepal	53.62	7.51
Armenia	53.26	7.31	Nigeria	62.28	12.67
Bangladesh	53.78	7.60	Panama	64.13	13.86
Belarus	49.75	5.34	Paraguay	64.19	13.90
Bhutan	56.73	9.31	Peru	60.93	11.83
Brazil	65.70	14.90	Philippines	59.11	10.72
Burkina Faso	57.62	9.83	Poland	53.24	7.30
Cambodia	55.84	8.79	Russian Federation	57.17	9.57
Chile	65.64	14.86	Rwanda	65.12	14.51
Colombia	66.98	15.77	Senegal	57.17	9.57
Costa Rica	63.87	13.69	Sierra Leone	55.16	8.39
Dominican Republic	61.37	12.10	Slovak Republic	50.34	5.67
Ecuador	62.02	12.51	South Africa	72.84	20.10
El Salvador	61.92	12.45	Sri Lanka	55.98	8.87
Ethiopia	54.24	7.87	Swaziland	63.98	13.76
Fiji	59.46	10.93	Tajikistan	52.12	6.66
Georgia	57.55	9.79	Thailand	57.17	9.57
Honduras	66.05	15.13	Togo	56.14	8.96
Indonesia	56.70	9.29	Tunisia	54.46	7.99
Jordan	55.16	8.39	Turkey	56.55	9.20
Kazakhstan	51.56	6.35	Uganda	60.37	11.49
Kyrgyz Republic	53.58	7.49	Ukraine	49.87	5.41
Latvia	53.97	7.71	Uruguay	59.89	11.19
Macedonia, FYR	58.21	10.18	Zambia	68.52	16.85
Madagascar	59.62	11.03	United States	74.10	21.13
Malawi	59.69	11.07			
Malaysia	60.10	11.32	Mean	57.76	10.63
Mali	53.29	7.33	Max	74.10	21.63
Mexico	61.54	12.21	Min	49.75	5.34
Moldova	53.34	7.35	Std. Dev	5.47	0.03

Figures and Tables

Figure 1: Ordering of Productivity Cutoffs



Note: Consumers within the same country endogenously consume different varieties of goods. High income consumers consume goods from lower productivity firms (z_{sd}^{H}), and therefore consume a larger variety than low income consumers. Low income consumers in an economy that does not have a binding resale constraint (equation 12) consume more than low income consumers in an equivalent economy where such a constraint binds ($z_{sd}^{L}<z_{sd}^{C}$).

Table 1: Parameter values for calibration

Parameter	Fact	Source
M_d	Population	WDI population
M_d^H	% of H-type population	40% of population
M_d^L	% of L-type population	60% of population
θ_d^H, θ_d^L	Income share and GDP	WDI Income Share, GDP, Model Solution
τ_{sd}	Share of imported trade value	Gravity Equation, Eaton and Kortum (2002)
N=55	# of countries in sample	-
γ=2.8	Pareto Shape	Simonovska and Waugh (2014)
b_d=1		Normalization
f_c=1		Normalization
q=1		Normalization
r	Regression Coefficent, $\beta^{v,s}$	Model Solution

Table 2: Regression Results: Differentiated Consumption Goods Trade and Inequality

	Trade Value		Trade Quantity		Unit Value	
$\log(CGDP_d)$	0.96***	0.78***	0.95***	0.75***	0.01	0.03**
	(0.02)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)
$\log(Pop_d)$	0.26***	0.25***	0.26***	0.25***	-0.00	-0.00
	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Inequality	–	0.08***	–	0.09***	–	-0.01***
	–	(0.00)	–	(0.00)	–	(0.00)
Product FE	Y	Y	Y	Y	Y	Y
Adj. R^2	0.50	0.55	0.62	0.63	0.83	0.83
Observations	13,535	13,535	13,535	13,535	13,535	13,535

***p<0.01; **p<0.05; *p<0.10

Table 3: Regression Results: Differentiated Non-Food Consumption Goods Trade and Inequality

	Trade Value		Trade Quantity		Unit Value	
$\log(CGDP_d)$	0.96***	0.76***	0.95***	0.73***	0.01	0.04**
	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)
$\log(Pop_d)$	0.28***	0.27***	0.28***	0.27***	-0.00	-0.00
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Inequality	–	0.09***	–	0.10***	–	-0.01***
	–	(0.00)	–	(0.00)	–	(0.00)
Product FE	Y	Y	Y	Y	Y	Y
Adj. R^2	0.51	0.53	0.58	0.60	0.77	0.77
Observations	11,810	11,810	11,810	11,810	11,810	11,810

***p<0.01; **p<0.05; *p<0.10

Figure 2: Values of r and Inequality Regression Coefficient

(a) Trade Value (b) Trade Quantity (c) Unit Value

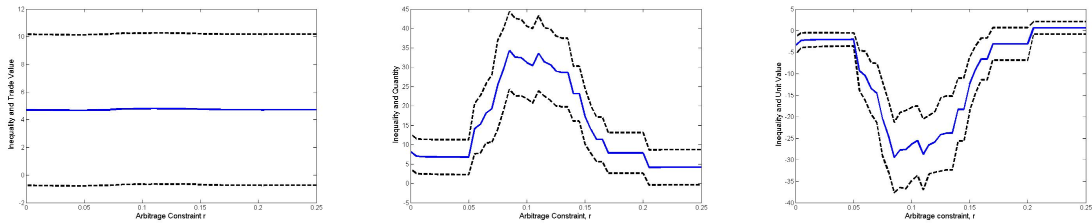


Table 4: Simulated Data Regression Results

	Data	$r = 0\%$	Unconstrained	Minimize Simulation Difference		
				GDPpc	Population	Inequality
Trade Value						
Optimal r value				Any	Any	9.30%
GDP per capita	0.76***	0.71***	0.71***	–	–	0.71***
Population	0.27***	0.58***	0.58***	–	–	0.58***
Inequality	0.09***	0.05*	0.05*	–	–	0.05*
Trade Quantity						
Optimal r value				10.0%	10.0%	15.8%
GDP per capita	0.73***	0.52***	0.49***	0.75***	0.75***	0.52***
Population	0.27***	0.18***	0.65***	0.28**	0.28**	0.67***
Inequality	0.10***	0.08***	0.04*	0.32***	0.32***	0.11***
Trade Unit Value						
Optimal r value				0.0%	–	5.3%
GDP per capita	0.04**	0.19***	0.23***	0.19***	–	0.24***
Population	-0.00	0.39***	-0.07***	0.39***	–	0.47***
Inequality	-0.01***	-0.03***	0.00	-0.03***	–	-0.02**

***p<0.01; **p<0.05; *p<0.10